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2年生冬期講習3日目のまとめ(問題の解答一挙掲載)

問題1 3つの解法で...

(2) $\Delta OAD = \frac{1}{2} \times OD \times 3 = \frac{1}{2} \times 6 \times 3 = 9$
 (3) $\Delta OAB = \frac{1}{2} \times (a+3) \times 6 = 15$
 (4) $\Delta OBC = \frac{1}{2} \times 4 \times OC = \frac{1}{2} \times 4 \times 6 = 12$

問題1 (1) 普通法...
 $A(-3, 9), B(2, 4)$
 AB の傾きは $\frac{4-9}{2-(-3)} = \frac{-5}{5} = -1$
 $y = -x + b$ とおく
 $A(-3, 9)$ を代入すると
 $9 = -(-3) + b \therefore b = 6$
 $\therefore y = -x + 6$

ABの傾きは $-3 + 2 = -1$
 y の切片は $-(-3) \times 2 = 6$
 $\therefore y = -x + 6$

$y = x^2 - (x+3)(x-2) = x^2 - x + 6$
 $y = -x + 6$

問題2 文字の計算に注意!

問題2
 $y = x^2$
 $A(a, a^2), B(a+1, (a+1)^2)$
 AB の傾きは $a+1$
 直線 AB の傾きは $-a$ とおく
 $\Delta OAB = \frac{1}{2} \times (1-a) \times OD = \frac{1}{2} \times (1-a) \times (-a) = \frac{1}{2} a(a-1)$

ABの式は $y = (a+1)x - a$
 $y = 0$ とおくと $(a+1)x - a = 0$
 $(a+1)x = a$
 $x = \frac{a}{a+1}$
 $\therefore C(\frac{a}{a+1}, 0)$
 $\Delta OBC = \frac{1}{2} \times OC \times 1 = \frac{1}{2} \times \frac{a}{a+1}$
 $\Delta OAB = \Delta OBC$ より $\frac{1}{2} a(a-1) = \frac{1}{2} \frac{a}{a+1}$
 $a-1 = \frac{1}{a+1}$
 $(a-1)(a+1) = 1$
 $a^2 - 1 = 1$
 $a^2 = 2$
 $a = \pm\sqrt{2}$
 $a < 0$ より $a = -\sqrt{2}$

問題3・問題4 今回のポイント問題

問題3
 $y = ax^2$
 (1) ABの式は $y = x + 6$
 (2) $P(t, t+6)$ とおく
 $\Delta OPB = 18$ より $\frac{1}{2} \times 6 \times t = 18$
 $t = 6$
 $\therefore P(6, 12)$
 $y = ax^2$ が P を通ると $12 = 36a \therefore a = \frac{1}{3}$

問題4
 $y = \frac{1}{4}x^2$
 $Q(4, -2)$
 $P(t, t+2)$
 $\Delta APB = 6$ より $\frac{1}{2} \times 4 \times (t-4) = 6$
 $2(t-4) = 6$
 $t-4 = 3$
 $t = 7$
 $\therefore P(7, 9)$
 $y = \frac{1}{4}x^2$ が P を通ると $9 = \frac{1}{4} \times 49$ (不適)

問題5 普通の解法と比較して!

問題5
 $y = \frac{1}{4}x^2$
 $A(-4, 4), B(n, \frac{1}{4}n^2)$
 AB の傾きは $\frac{\frac{1}{4}n^2 - 4}{n - (-4)} = \frac{n^2 - 16}{4(n+4)} = \frac{n-4}{4}$
 $y = \frac{1}{4}(n-4)x + b$ とおく
 $A(-4, 4)$ を代入すると $4 = \frac{1}{4}(n-4)(-4) + b$
 $4 = -(n-4) + b$
 $\therefore b = n$
 y の切片は n

(1) 直線 AP の傾きは $\frac{1}{4}(n-4)$
 直線 AP の傾きは $-\frac{1}{4}(n-4)$ とおく
 $n = \frac{1}{4}(n-4)$
 $4n = n-4$
 $3n = -4$
 $n = -\frac{4}{3}$
 $\Delta APQ = \frac{1}{2} \times 4 \times n = 2n = -\frac{8}{3}$

(2) $l = 9 - (n-4) = 13 - n$
 $h = \frac{1}{4}(n-4) + \frac{1}{4}(13-n) = \frac{1}{4}(n-4+13-n) = \frac{1}{4}(9) = \frac{9}{4}$
 $\Delta APQ = \frac{1}{2} \times 4 \times \frac{9}{4} = 9$

問題6 比の保存・解の公式

問題6
 $y = x^2$
 $A(a, a^2), B(a-2, (a-2)^2)$
 AB の傾きは $a-2$
 $y = (a-2)x - a$ とおく
 $y = 0$ とおくと $(a-2)x - a = 0$
 $(a-2)x = a$
 $x = \frac{a}{a-2}$
 $\therefore C(\frac{a}{a-2}, 0)$
 $\Delta OBC = \frac{1}{2} \times OC \times (a-2) = \frac{1}{2} \times \frac{a}{a-2} \times (a-2) = \frac{1}{2} a$

(2) $PB : BA = 1 : 2$ とおく
 $PB' : BA' = 1 : 2$ とおく
 $P(t, t)$ とおく
 $(a-2-t) : (a-a+2) = 1 : 2$
 $(a-2-t) : 2 = 1 : 2$
 $2(a-2-t) = 2$
 $a-2-t = 1$
 $t = a-3$
 $OC = \frac{a(a-2)}{2a-2}$

ABの式は $y = (2a-2)x - a(a-2)$
 $2(a-1)(a-1) = a(a-2)$
 $2(a^2 - 2a + 1) = a^2 - 2a$
 $2a^2 - 4a + 2 = a^2 - 2a$
 $a^2 - 2a + 2 = 0$
 $a = 3 \pm \sqrt{3}$
 $1 < a < 2$ より $a = 3 - \sqrt{3}$

問題7 回転体の体積

問題7
 $y = x^2$
 $A(-2, 4), B(2, 4)$
 AP の傾きは $t-2$
 AP の傾きは $-2t$
 $t-2 = -2t$
 $3t = 2$
 $t = \frac{2}{3}$
 $\therefore P(\frac{2}{3}, \frac{4}{9})$

(2) ΔOAB が直角二等辺三角形
 $OC = h$
 $h = l$
 $\therefore l = 2l^2$
 $l = \frac{1}{2}$
 $A(\frac{1}{2}, \frac{1}{4}), B(\frac{1}{2}, \frac{1}{4})$
 $\therefore \Delta OAB$ が正三角形
 $h = \frac{\sqrt{3}}{2} l$
 $\therefore \frac{\sqrt{3}}{2} l = 2l^2$
 $l = \frac{\sqrt{3}}{2}$
 $A(\frac{\sqrt{3}}{2}, \frac{3}{4}), B(\frac{\sqrt{3}}{2}, \frac{3}{4})$

問題8 2乗比例法則

問題8
 $y = x^2$
 $A(2, 1), B(4, 4)$
 AB の傾きは 1
 $y = x - 1$ とおく
 $y = 0$ とおくと $x - 1 = 0$
 $x = 1$
 $\therefore C(1, 0)$
 $\Delta OBC = \frac{1}{2} \times 3 \times 4 = 6$

(2) ΔOAB が正三角形
 $OC = h$
 $h = l$
 $\therefore l = 2l^2$
 $l = \frac{1}{2}$
 $A(\frac{1}{2}, \frac{1}{4}), B(\frac{1}{2}, \frac{1}{4})$
 $\therefore \Delta OAB$ が正三角形

問題9 2乗比例法則

問題9
 $y = \frac{1}{4}x^2$
 $A(2, 1), B(4, 1)$
 AB の傾きは 0
 $y = 1$ とおく
 $y = \frac{1}{4}x^2$ が $y = 1$ を通ると $1 = \frac{1}{4}x^2$
 $x^2 = 4$
 $x = \pm 2$
 $\therefore C(-2, 1), D(2, 1)$
 $\Delta OAB = \frac{1}{2} \times 2 \times 1 = 1$
 $\Delta OBC = \frac{1}{2} \times 4 \times 1 = 2$
 $\Delta OAD = \frac{1}{2} \times 4 \times 1 = 2$
 $\Delta OAB = \Delta OBC = \Delta OAD$

(2) $\square PQSR$ が正方形
 $2a = \frac{1}{4}a^2$
 $a^2 = 8a$
 $\therefore a = 8$ ($a=0$ は不適)

問題10 放物線の対称性

問題10
 $y = x^2 - 2x + 2$
 $A(2, 1), B(4, 4)$
 AB の傾きは 1
 $y = x - 1$ とおく
 $y = 0$ とおくと $x - 1 = 0$
 $x = 1$
 $\therefore C(1, 0)$
 $\Delta OBC = \frac{1}{2} \times 3 \times 4 = 6$

(2) ΔOAB が正三角形
 $OC = h$
 $h = l$
 $\therefore l = 2l^2$
 $l = \frac{1}{2}$
 $A(\frac{1}{2}, \frac{1}{4}), B(\frac{1}{2}, \frac{1}{4})$
 $\therefore \Delta OAB$ が正三角形

問題11 放物線の対称性

問題11
 $y = ax^2 + b$
 $A(1, a+b), B(2, 4a+2b)$
 AB の傾きは $3a$
 $y = 3ax + b$ とおく
 $y = 0$ とおくと $3ax + b = 0$
 $x = -\frac{b}{3a}$
 $\therefore C(-\frac{b}{3a}, 0)$
 $\Delta OBC = \frac{1}{2} \times \frac{b}{3a} \times (4a+2b) = \frac{b(4a+2b)}{6a}$

(1) $y = ax^2 + b$ が $A(1, a+b)$ を通ると $a+b = a+b$
 $\therefore a+b = 0$
 $b = -a$
 $\therefore y = a(x^2 - 1)$
 $A(1, 0), B(2, 0)$
 $\Delta OAB = 0$

(2) $y = ax^2 + b$ が $A(1, a+b)$ を通ると $a+b = a+b$
 $\therefore a+b = 0$
 $b = -a$
 $\therefore y = a(x^2 - 1)$
 $A(1, 0), B(2, 0)$
 $\Delta OAB = 0$